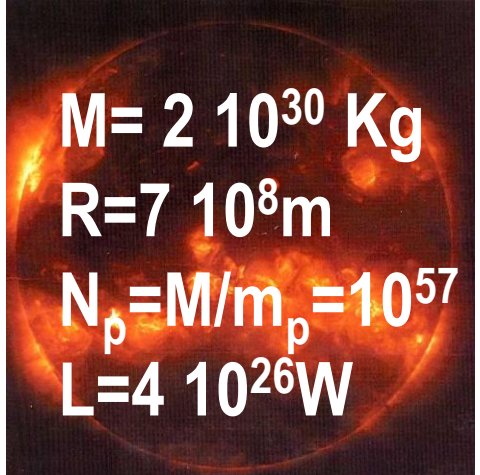


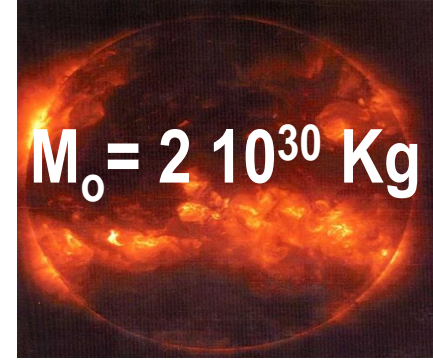
# A first look at the Sun



$M = 2 \cdot 10^{30} \text{ Kg}$   
 $R = 7 \cdot 10^8 \text{ m}$   
 $N_p = M/m_p = 10^{57}$   
 $L = 4 \cdot 10^{26} \text{ W}$

- Observables about global properties:
  - Mass, Luminosity, Radius,
  - Metal content of the photosphere
  - Age
- A first look at the solar interior: helioseismology
- Typical scales :density, pressure, sound speed, temperature
- Energy production in the Sun: where does it come from?

# Solar mass in Astronomy and Astrophysics



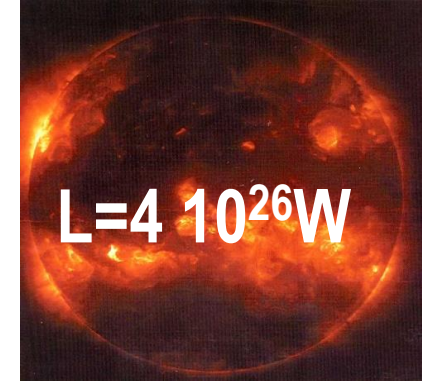
- Astronomy (i.e. study of orbit properties) only deals with the extremely well determined Gaussian constant:

$$G_N M_{\odot} = (132\,712\,438 \pm 5) \cdot 10^{12} \text{ m}^3/\text{s}^2$$

- Astrophysics needs  $M_{\odot}$ , since physical properties are determined by the number of electrons and nucleons in the star:
  - opacity is determined by
$$N_e \approx N_N \approx M_{\odot}/m_p \approx 10^{57}$$
  - energy content/production of the star depends on  $N_N$ .
- Cavendish by determining  $G_N$  provided a measurement of  $M_{\odot}$ . The (poor) accuracy on  $G_N$  (relative error 0.15%) reflects that on  $M_{\odot}$ :

$$M_{\odot} = 1.989 (1 \pm 0.15\%) \cdot 10^{33} \text{ gr} \approx 10^{57} m_p$$

# Solar Luminosity



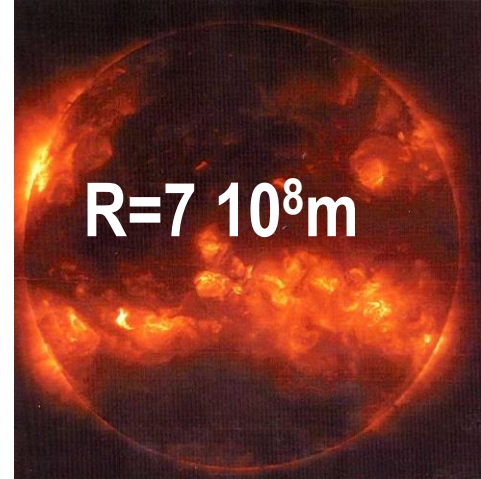
- Derived from measurements of the “solar constant”  $K_o$ : the amount of energy per unit time and unit area, impinging from the Sun onto Earth  $K = 1.4 \text{ KW/m}^2$  \*.
- Not precisely a constant, it varies with time (0.1% in a solar cycle). The average over 12 years of solar irradiance (and over different satellite radiometers) multiplied by Sun Earth distance, gives the solar luminosity:

$$L_o = 4\pi d^2 K_o = 3.844(1 \pm 0.4\%) 10^{26} \text{ W}$$

( $L_o \approx 10^{17}$  nuclear power plants)

\*see [http://en.wikipedia.org/wiki/Solar\\_constant#Solar\\_constant](http://en.wikipedia.org/wiki/Solar_constant#Solar_constant)

# Solar Radius



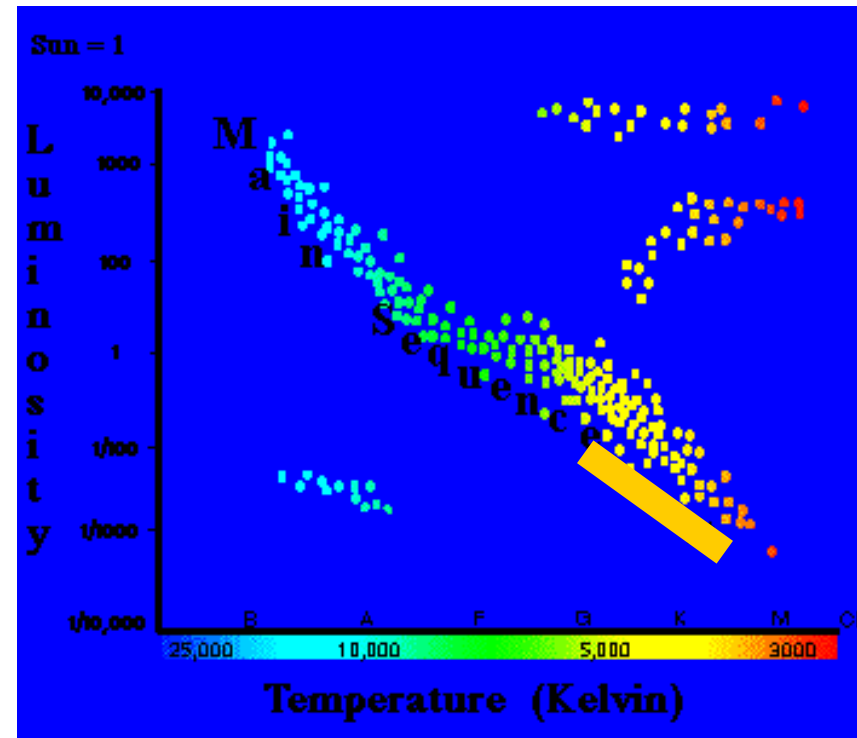
- The distance from the center of the sun to its visible surface (the photosphere)
- Difficult to define the edge of the sun
- Different methods and different experiments:  $R_{\odot}=6.9598(1\pm 0.04\%) 10^{10}$  cm
- In astronomy, the solar radius is used as a unit of length used to express the size of stars. It is equal to the current radius of the Sun. Its value is: about 110 times the radius of the Earth, or 10 times the average radius of Jupiter.
- It varies slightly from pole to equator due to its rotation, which induces an oblateness of order 10 parts per million



# Solar Age

- Method: radioactive dating of oldest objects in the solar system (chondritic meteorites)
- Problems:
  - relationship between the age of the meteorite and the age of the sun
  - what is the zero time for the sun?
- The age of the sun referred to Zero Age Main Sequence

$$T=4.57(1 \pm 0.4\%) \text{ Gyr}$$



# Solar Metal abundance

- Spectroscopic measurements of the solar photosphere yield the relative abundances (in mass) of “metals“ to H
$$(Z/X)_{\text{photo}} = 0.0233(1 \pm 6\%)$$
- Most abundant metals: O, C, N, Fe
- Results are generally consistent with the meteoritic abundances
- A remarkable exception: the solar Li content is depleted by 100 with respect to meteorites

Note: Hydrogen abundance  $X \sim 0.75$

# A remark: Helium abundance

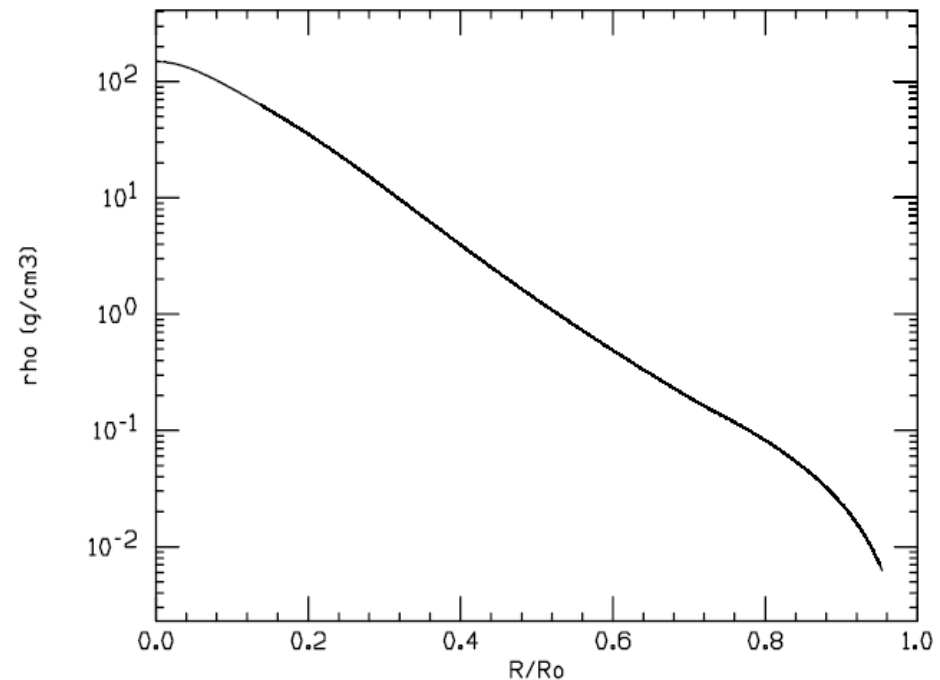
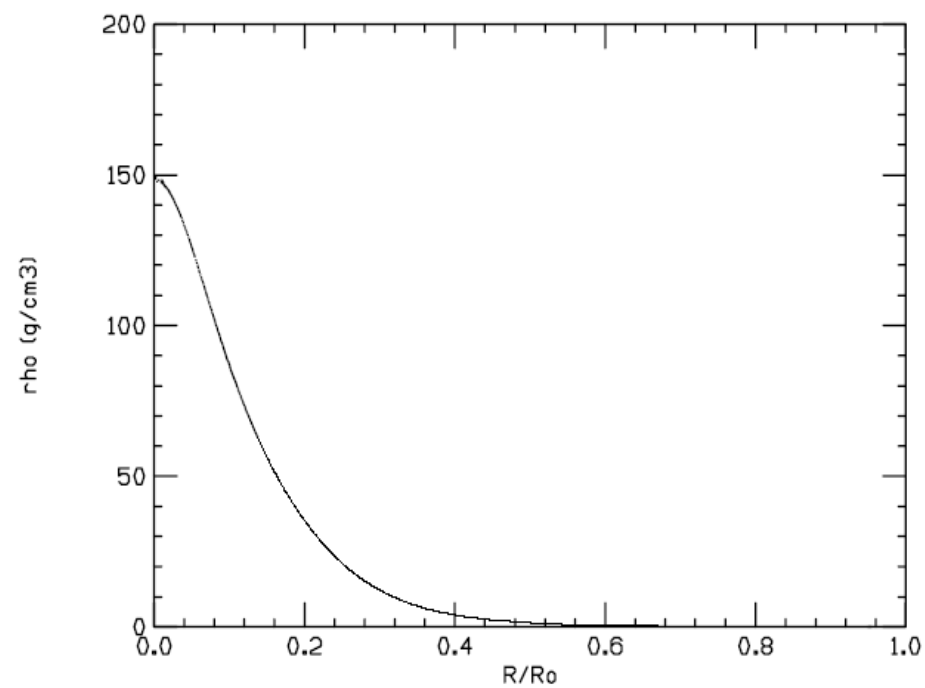
- Helium was discovered in the Sun (1895), but its abundance cannot be accurately measured there.
- Until a few years ago, estimate of the photospheric He abundance was taken the result of solar models
- Helioseismology provides now an indirect measurement.....

# Typical scales: density

- The average solar density is :

$$\rho = 3M_{\odot}/(4\pi R_{\odot}^3) \approx 1.5 \text{ g/cm}^3$$

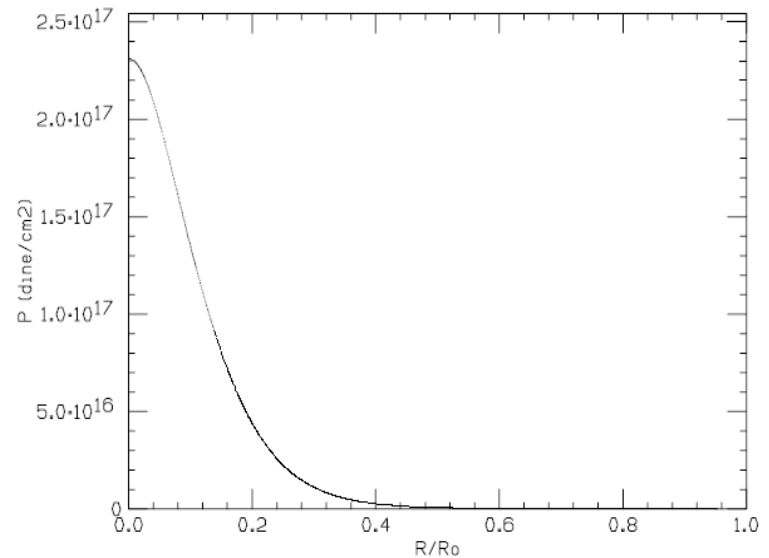
- The (calculated) density profile shows that density is strongly varying, reaching  $150 \text{ g/cm}^3$  near the solar centre.



# Typical scales:

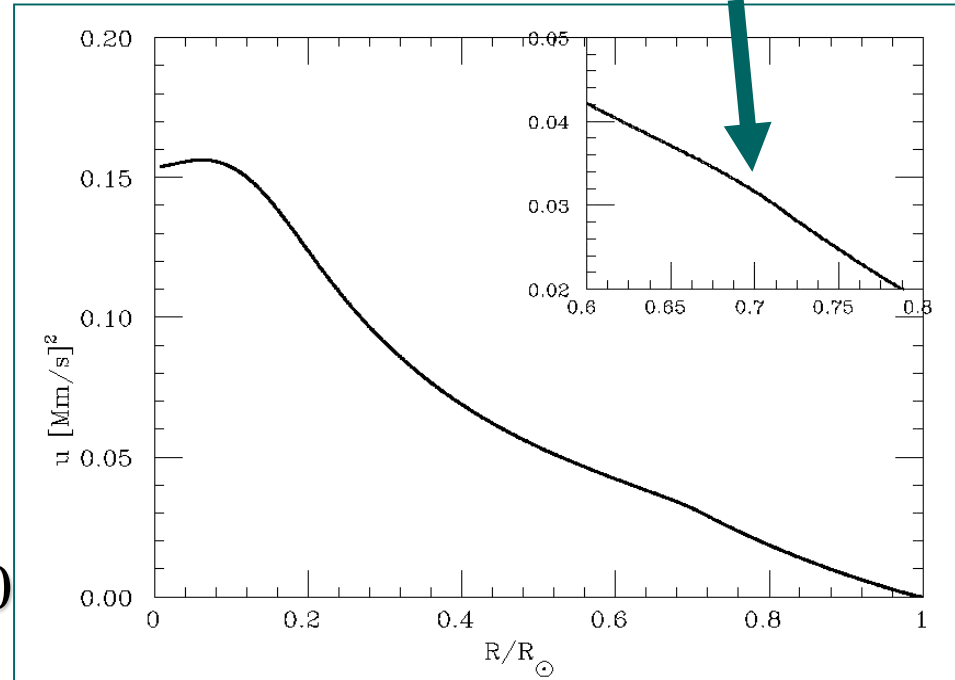
## Pressure

- By dimensional arguments one finds the typical scale of pressure  
 $[P] = [F]/[S]$  ;
- With the physical parameters of the Sun one has  $F \approx GM_{\odot}^2/R_{\odot}^2$  and  $S \approx R_{\odot}^2$ . This gives  
 $P \approx GM_{\odot}^2/R_{\odot}^4 \approx 10^{16}$  dine/cm<sup>2</sup>
- This corresponds to  $10^{15}$  Pascal or  $10^{10}$  Atm
- Pressure, of course is greatest at the center and vanishing on the surface, see the (calculated) solar profile



# Typical scales: Sound speed

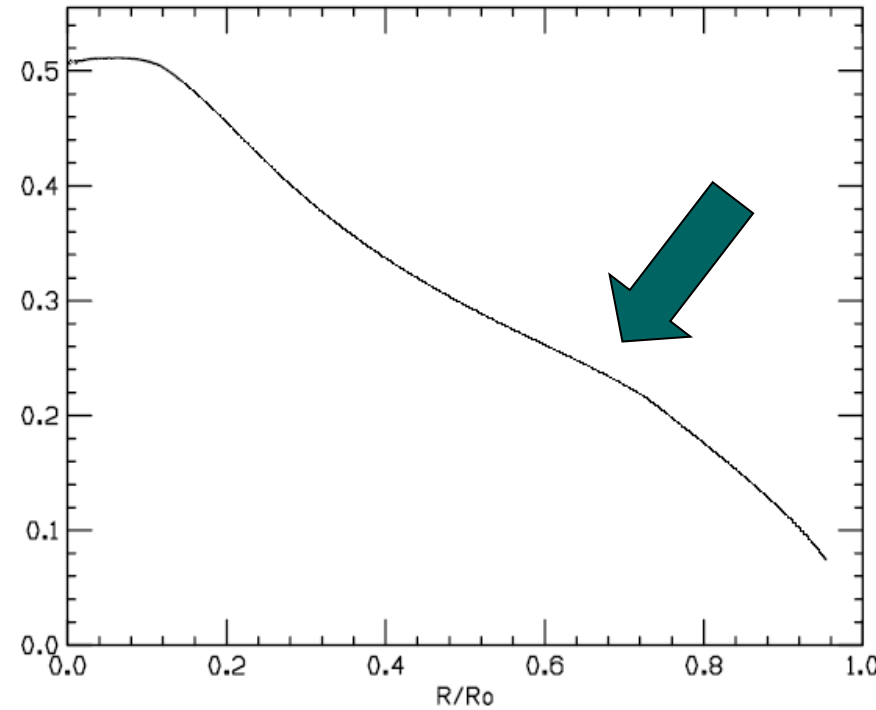
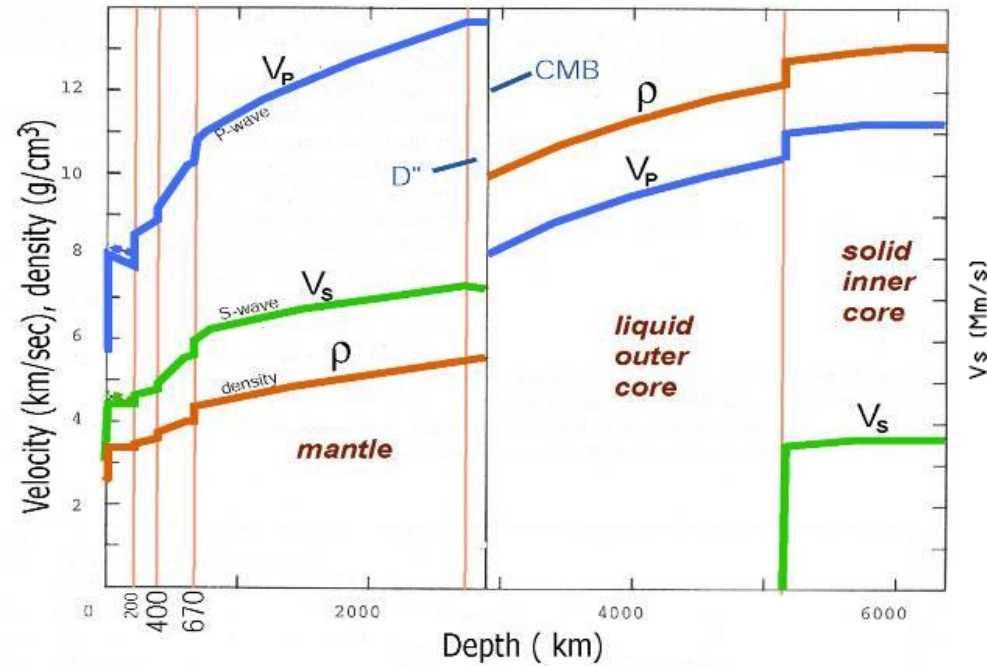
- Again by dimensional arguments  
 $v_s \approx (P/\rho)^{1/2} \approx 10^6 \text{ m/s}$
- Just as an example, compare with sound speed in air in the lab  $\approx 300 \text{ m/s}$
- The time for a sound wave to propagate through the Sun is  
 $t_s \approx R/v_s \approx 10^3 \text{ s}$
- The acoustic frequencies are thus of order  $\nu = 1/t_s \approx \text{mHz}$ .
- The figure shows the **reconstructed** sound speed along the solar profile.



- Note the knee near  $0.7 R_{\odot}$  signalling some change in the solar structure (transition from radiative to convective transport)
- Note that a sound speed profile is physically different from a temperature profile: the missing information is composition



# Sound speed in the Earth and in the Sun



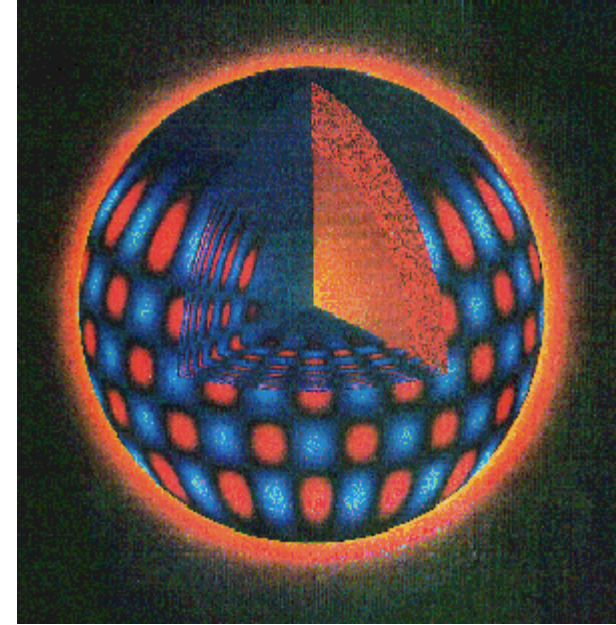
- Earth has a layered structure with marked discontinuities.
- Both transverse and longitudinal waves propagate in the solid part

- Sun is a gaseous sphere. No discontinuity in sound speed or density, but only a discontinuity in first derivative, corresponding to a change of energy transport

# How to look into the solar interior?

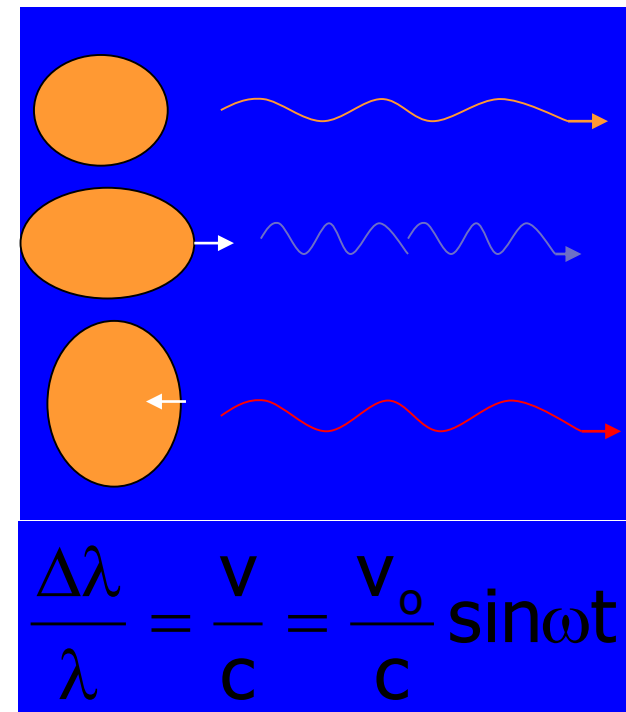
## Helioseismology

- As one studies the deep Earth's structure through earthquakes or just like you can tell something about a material by listening to the sounds that it makes when something hits it, so one can study the solar interior by looking at its vibrations.
- Birth: in 1960 it was found that the solar surface vibrates with a period  $T \approx 5$  min,
- Plan: reconstruct the properties of the solar interior by studying how the solar surface vibrates



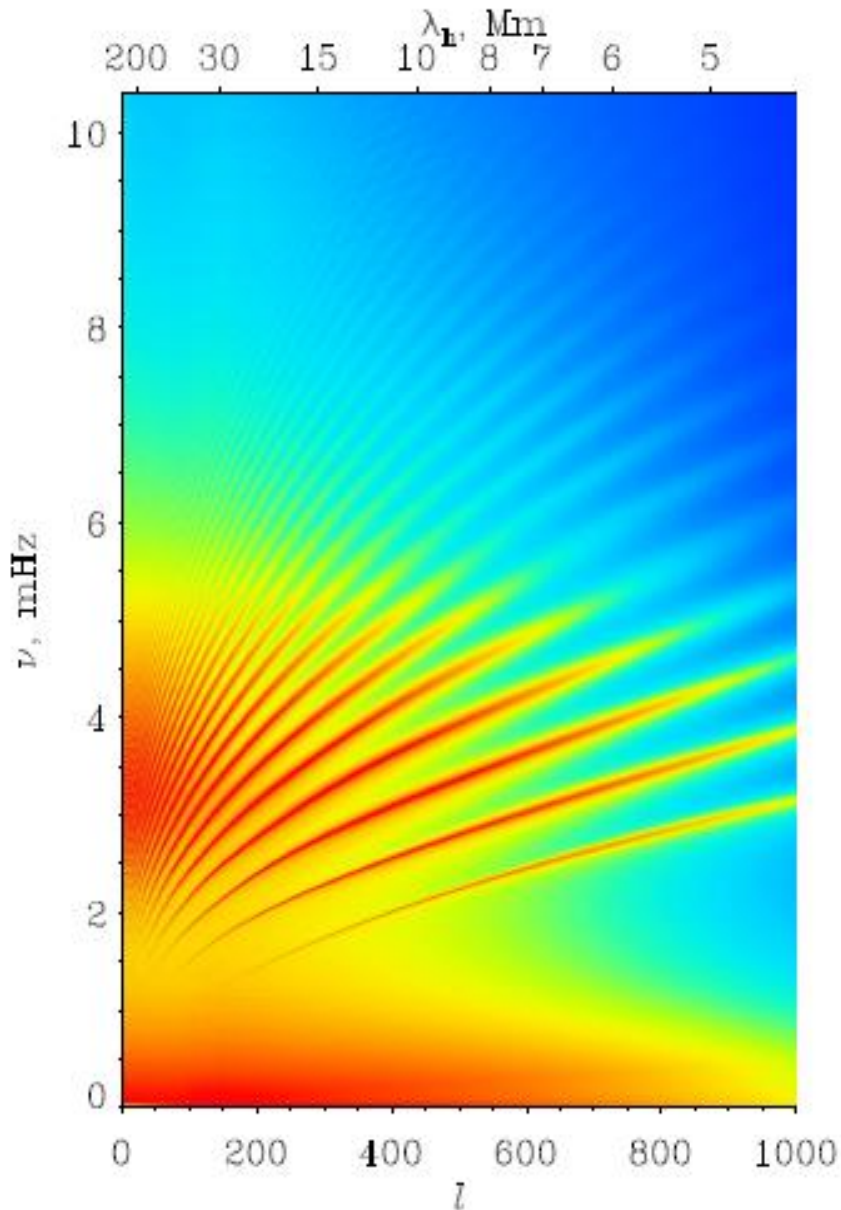
# Method

- By means of Doppler effect on the emitted radiation, one can measure oscillations of the solar surface with a very high accuracy
- Most recent measurements performed with SOHO satellite (SOlar and Heliospheric Observatory)



<http://sohowww.nascom.nasa.gov/>

# The modes

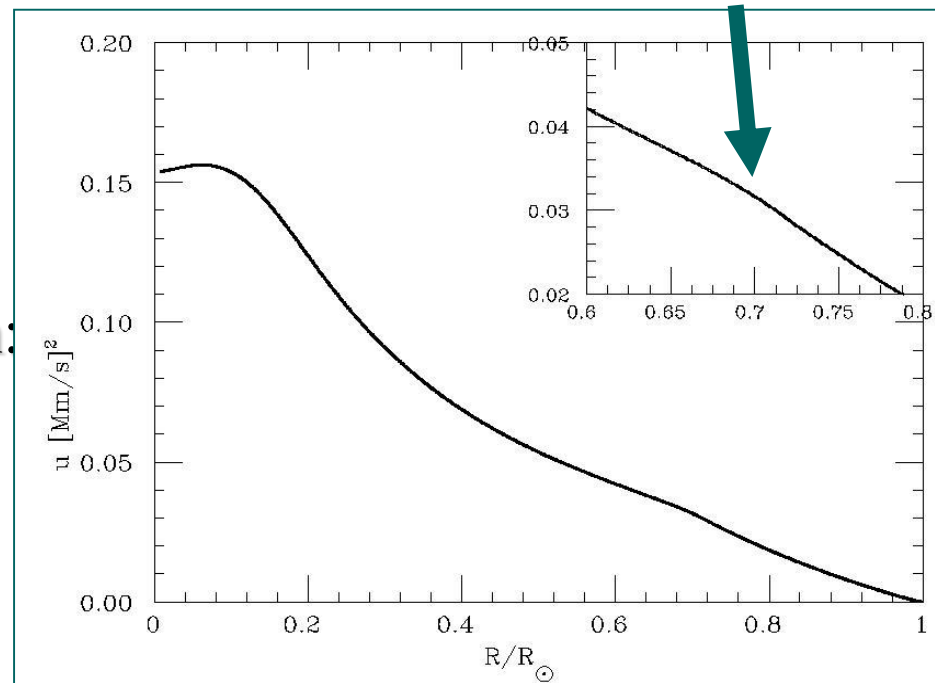


- The observed oscillations are decomposed into discrete modes.
- Note the mHz scale; color corresponds to intensity.
- Some  $10^4$  modes are available
- Their frequencies are accurately determined  
 $(\Delta\omega/\omega \approx 10^{-3} - 10^{-4})$
- So far, only p-modes have been observed, i.e. pressure driven oscillations.

# Helioseismic inferences

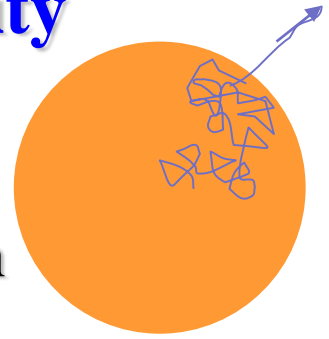
By comparing the measured frequencies with the calculated ones (inversion method) one can determine:

- The sound speed profile (with accuracy of order 0.5% )
- Locate the transition between radiative transport and convection:  
 $R_b = 0.711 (1 \pm 0.14\%) R$
- The photospheric He abundance:  
 $Y_{\text{photo}} = 0.249 (1 \pm 1.4\%)$
- Provide a benchmark for solar model builders.





# Typical scales: photon mean free path and opacity



- In the bulk of the Sun, energy is transported by radiation
- Photons will propagate with a mean free path  $\lambda$ .
- As an order of magnitude estimate;

$$\lambda = 1/(n_e \sigma_{Th}) \approx 1/(10^{24} \text{ cm}^{-3} 10^{-24} \text{ cm}^2) \approx 1 \text{ cm}$$

- Note that it takes some  $10^4$  years for photons to cross the Sun (exercise), since for brownian motion the travelled distance  $D$  is related to mean free path, velocity and time by:

$$D = (\lambda v t)^{1/2}$$

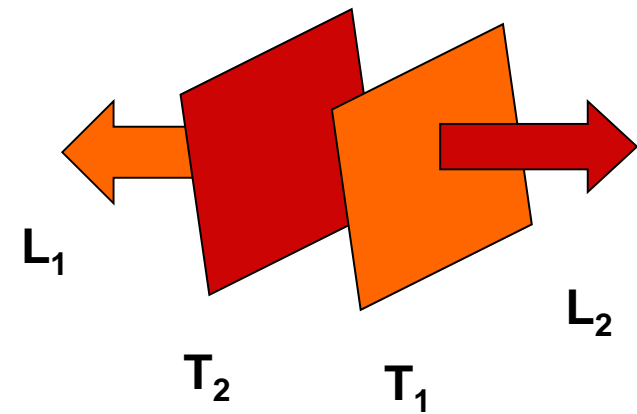
- Opacity is defined as  $\kappa = 1/\lambda$  and thus  $\kappa \approx 1 \text{ cm}^{-1}$ .



# Energy transport by radiation

- Remind that black body at temperature  $T$  radiates energy per unit area and time given by the Stefan-Boltzmann law:

$$L = \sigma T^4$$



- In order to transport energy a temperature difference is needed
- Consider a temperature gradient between 1 and 2 ( $T_1 < T_2$ ). More energy flows from 2 to 1 than in the opposite direction.
- The net energy transported  $\Phi$ , per unit area and time, is given by the difference in energy radiated (per unit area and time) by two black bodies separated by  $\lambda$ ,  $\Phi = L_2 - L_1$

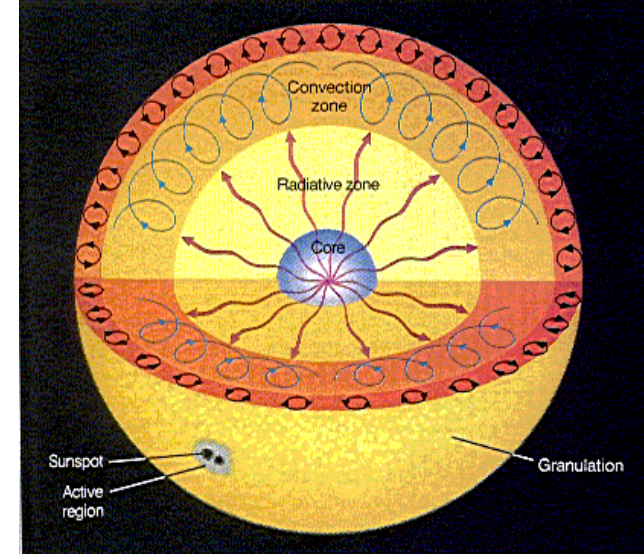
One can expand the difference to lowest order in the temperature difference and obtain:

$$\Phi = \lambda \nabla \sigma T^4 = 4 \sigma / \kappa T^3 \nabla T$$

- This is the equation for radiative energy transport, which shows the role of opacity in energy transport

# Energy transport in the Sun: radiation and convection

- The energy transport by radiation is based on the action of photons, which are generally few in number as compared with the number of particles available in a gas (exercise). If matter moves, it brings heat with it and this transport mechanism is much more efficient.
- Matter is in hydrostatic equilibrium ( $\nabla P = -\rho \mathbf{g}$ ) but equilibrium becomes unstable and convection starts when temperature gradient is large enough: Schwarzschild has shown the condition is:  $dT/dz > g/C_p$  where  $g$  is the gravitational acceleration in the medium and  $C_p$  is the specific heat at constant Pressure.
- This is what occurs in the Sun at about 70 %  $R_{\odot}$ .
- In the solar interior energy propagates by radiation (radiative region) whereas in the outer region (convective envelope) energy is transported by convection.



# Typical scales: Energy

- The cumulated energy production over the Sun history can be estimated assuming that the present solar luminosity  $L_{\odot}$  has remained constant over the solar age

$$\Delta E = L_{\odot} t \approx 10^{44} \text{ Joule}$$

- The energy equivalent of the Sun (rest energy) is:

$$M_{\odot} c^2 \approx 10^{47} \text{ Joule}$$

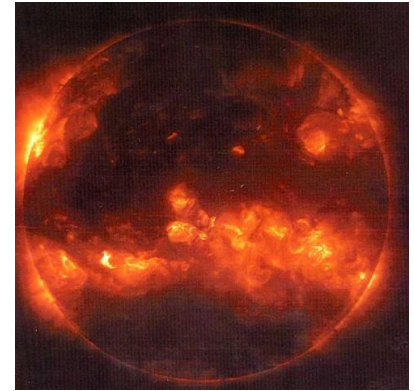
- Thus cumulated emitted energy over rest energy of the Sun is:

$$\Delta E / M_{\odot} c^2 \approx 10^{-3}$$

- Which is typical of a nuclear process, where 1 MeV is released in reaction involving GeV mass particles

$$\varepsilon_{\text{nuc}} = \frac{\Delta E_{\text{nuc}}}{m c^2} \approx \frac{1 \text{ MeV}}{1 \text{ GeV}}$$

# Sun energy inventory



- The present heat flow  $L$  can be sustained by an energy source  $U$  for an age  $t$  provided that  $U > Lt$  :

		[yr]
a)chemistry:	$U \approx (1\text{eV})N_p$	$\rightarrow t_{\text{ch}} = 2 \cdot 10^4$
b)gravitation	$U \approx GM^2/R$	$\rightarrow t_{\text{gr}} = 3 \cdot 10^7$
c)nuclear	$U \approx (1\text{MeV})N_p$	$\rightarrow t_{\text{nu}} = 2 \cdot 10^{10}$

Only nuclear energy can sustain the Solar luminosity over the sun age,  
 $t = 4.5 \cdot 10^9$  y

**Can we prove that actually the Sun shines due to nuclear energy ?**

# A digression:

## The Earth energy inventory

It is not at all easy to understand the dominant contribution to the Earth energy production.

- The present heat flow  $H_E = 40$  TW can be sustained by an energy source  $U$  for a time  $t = U/H_E$  :
- a) "chemistry"\*):  $U = (0.1 \text{ eV})N_p = 6 \cdot 10^{31} \text{ J}$   $\rightarrow t_{ch} = 5 \cdot 10^{10} \text{ y}$
- b) gravitation  $U = GM^2/R = 4 \cdot 10^{32} \text{ J}$   $\rightarrow t_{gr} = 3 \cdot 10^{11} \text{ y}$
- c) nuclear \*\*)  $U = (1 \text{ MeV})N_{p,rad} = 6 \cdot 10^{30} \text{ J}$   $\rightarrow t_{gr} = 5 \cdot 10^9 \text{ y}$
- Thus all energy sources seem capable to sustain  $H_E$  on geological times.

\*) actually it means the solidification (latent) heat

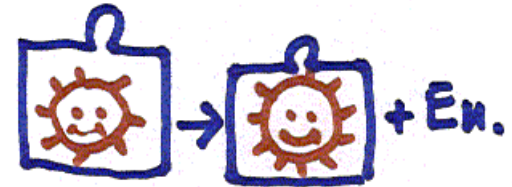
\*\*) the amount of radioactive material is taken as  $M_{rad} \approx 10^{-8} M_{Earth}$



# The birth of Nuclear Astrophysics

Eddington: Nature (1920)

“Certain physical investigations in the past year make it probable to my mind that some portion of sub-atomic energy is actually being set free in a star. ... If five per cent of a star's mass consists initially of hydrogen atoms, which are gradually being combined to form more complex elements, the total heat liberated will more than suffice for our demands, and we need look no further for the source of a star's energy...”



$$\Delta E = L_0 t \approx 10^{-3} M_0 c^2$$

• In the same paper: “If indeed the sub-atomic energy in the stars is being freely used to maintain their great furnaces, it seems to bring a little nearer to fulfillment our dream of controlling this latent power for the well-being of the human race - or for its suicide”

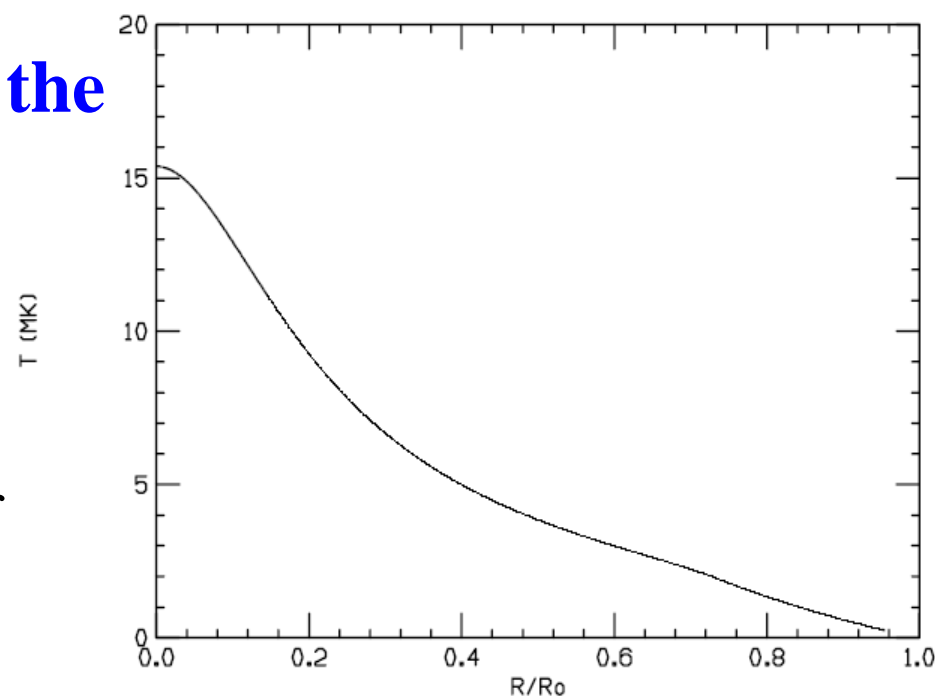


# Nuclear reactions in the Sun: the scale of Temperature

- An equation of state is needed in order to connect  $P$  and  $\rho$  with temperature  $T$ .
- Assume this is given by the eq. of classical, perfect gases for fully ionized Hydrogen;

$$P = nKT = (n_e + n_p)kT = 2(\rho/m_p) kT$$

- This can be used to derive the typical scale of  $T$   
 $kT \approx m_p (P/\rho) \text{ keV} \approx 10^7 \text{ K}$
- This is much larger than the temperature of the photosphere,  $\approx 6000 \text{ K}$
- The calculate profile of Temperature shows that in the centre it reaches  $T_c = 1.52 \cdot 10^7 \text{ K}$
- Solar model builders claim to know this number with an accuracy of about 1 %



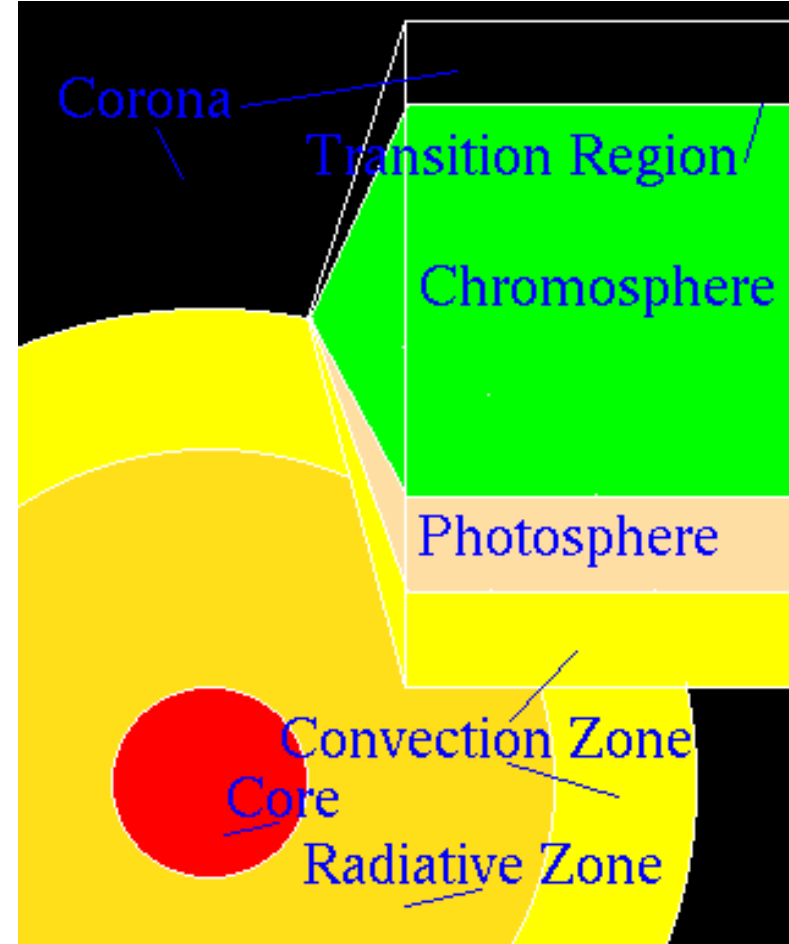
# Nuclear reactions in the Sun?

## The Core

- The Sun core is defined as the region where nuclear reactions sustaining the solar power occur
- It extends up to 10% of  $R_{\odot}$ , it contains some 1/3 of the solar mass
- Density is of order  $100 \text{ g/cm}^3$
- Temperature is of order of  $10^7 \text{ K}$ , corresponding to kinetic energies of order keV
- Note there is no boundary between the core and the rest of the radiative region
- Nuclei in the Sun have to overcome the Coulomb repulsion, which occurs through tunnel effect, see chapter 1
- Only the hydrogen nuclei can react, to form  $4\text{He}$  nuclei

# The gross solar structure

- Core  
 $R < 0.1 R_{\odot}$        $M \approx 1/3 M_{\odot}$   
(the site of nuclear reactions)
- Radiative zone  
 $(0.1 \div 0.7) R_{\odot}$        $M \approx 2/3 M_{\odot}$
- Convective zone  
 $(0.7 \div 1) R_{\odot}$        $M \approx 2\% M_{\odot}$   
(As temperature drops, opacity increases and radiation is not efficient for energy transport)
- Photosphere  
the very tiny layer of the Sun that we can observe directly



# A first summary

- The knowledge of  $M$ ,  $R$ ,  $L$  and age immediately determines the physical scales ( $T$ ,  $P$ ,  $\rho$ ) of the solar interior and shows that nuclear reactions are the only sufficient source of solar power.
- Helioseismology provides an accurate picture of the solar interior. It measures sound speed, not temperature
- The only source which can sustain the Sun over its age is nuclear energy.
- We need a proof that nuclear energy actually sustains the Sun

# How can we look at the solar interior?

- *“If there are more things in heaven and Earth than are dreamt of in our natural philosophy, it is partly because electromagnetic detection alone is inadequate”*
- We have two probes:
  - Sound waves (already discussed)
  - **Neutrinos** (see next chapters)

